Mathematics Curriculum in Singapore

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Abstract: The mathematics curriculum in Singapore consists of a set of syllabus started from primary school to pre-university for 12 years. The syllabus at the primary school level is aimed to create a strong foundation, while at the secondary school level, the syllabus is aimed to build strength, and at the pre-university level, it is intended to prepare for further education. The final goal of the mathematics curriculum in Singapore is problem-solving ability. To achieve this problem-solving ability, five components are created, namely: concepts, skills, thought processes, metacognition, and interrelated attitudes. The syllabus for each level is formed by 3 mathematical concepts and skills, as well as related to the learning experience (process, metacognition and attitude).

Introduction

Mathematics is one of the compulsory subjects that must be studied and Mastered by students at various levels of Education. Likewise, in Singapore, mathematics courses are given at the primary and secondary school levels, while at the pre-university level it is optional [1]. Mathematics contributes to the development and understanding of various sciences that underlie today's innovations, provide solutions for the future, describe and understand life phenomena, create lifestyle products and techniques, improve productivity, decision making, and Safety [2]–[8]. The use of mathematics in life, including: the provision of goods that match the interests, preferences, needs, and lifestyle of consumers; knowing population growth; predicting the spreading of disease outbreaks that hit; business analysis, market surveys, academic research, etc.

The mathematics curriculum in Singapore consists of a set of syllabus used from primary school to pre-university for 12 years. In outline, the mathematics curriculum is divided into 3 parts, namely: mathematics curriculum for elementary, secondary, and pre-university levels. The elementary level mathematics curriculum is aimed to create a strong foundation, the intermediate level is aimed to build strength, and the pre-university level is aimed to equip and prepare students for university [9]. The general objectives of the mathematics curriculum are: a) to enable students to acquire and apply mathematical concepts and skills; b) to develop cognitive and metacognitive skills through a problem-solving approach; and c) to develop a positive attitude [9], [10]. To get know more an idea of how the mathematics curriculum in Singapore, the following will be discussed about how the mathematics syllabus, curriculum framework, strands on the syllabus, approach in building strands, and its implementation.

Discussion

Mathematics Syllabus

In particular, each syllabus of each level has a different purpose depending on the needs of the content and skills of each level. The mathematics curriculum at the elementary school level aims to: 1) equip students with mathematical concepts and skills used in learning mathematics and everyday life; 2) Develop reasoning, communication, applicative, and metacognition skills for problem solving; and 3) build self-confidence and foster interest in mathematics [9]. At the secondary school level aims to: 1) ensure that all students achieve a level of mastery of mathematics that is useful and functions effectively in everyday life; and 2) provide provision for students who have more interest and ability in studying mathematics to be able to continue the study of Mathematics or other studies related to mathematics at the next level [4], [7], [8].

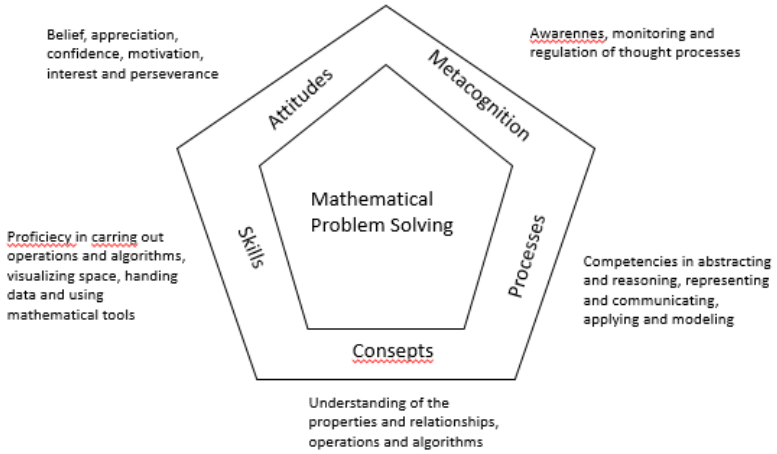
While the goal at the pre-university level depends on the needs of each syllabus at that level. There are 4 syllabus at the pre-university level, namely: Mathematics syllabus H1, H2, advanced H2, and H3. The H1 Mathematics syllabus provides the mathematical foundations and statistical methods that will support the next level of study in business or social sciences [5]; the H2 Mathematics syllabus prepares students in a wide range of areas at the university, including Mathematics, Science, and areas that require a good foundation of mathematics [6]. The advanced H2 Mathematics syllabus is designed to equip students who will expand and deepen their knowledge of mathematics and its applications [2]. For H3 Mathematics syllabus is designed for students who have interest and ability in mathematics and are interested in specialising and verificating in mathematics [3].

The mathematics syllabus 2020 has the following keywords, namely: 1) developing critical thinking, reasoning, communication and modeling processes that can enhance mathematical learning and support the development of 21st century competencies; 2) developing awareness of the nature and

mathematical ideas that become the main of the discipline and bring coherence and connection between different topics, as well as develop a deeper and stronger understanding; and 3) paying attention in developing metacognition by promoting the self-learning and self-reflection [4], [7], [8].

Mathematics Curriculum Framework

The curriculum structure serves as a framework that provides context and influences other categories [11]. Direction setting and implementation guidance in mathematics learning and assessment refers to a framework that focuses on developing mathematical problem-solving competencies [12], [13]. The development of problem-solving competencies is supported by the interrelationships among the components of concepts, skills, processes, metacognition and attitudes. The mathematics curriculum framework in Singapore can be presented in Figure 1 below.



Picture 1. Mathematics curriculum framework in Singapore [2], [3], [5], [6]

According to the Ministry of Education Singapore [2], [3], [5], [6], [9] strands are organized by content and strands are interconnected and related each other. In other different syllabuses there are variations on the vastness and profunity of the content. For example, at the elementary school level, it focuses on 3 concepts which include: numbers and algebra, measurement and geometry, and statistics. High school focuses on numbers and algebra, geometry and measurement, and statistics and probability. Math skills are very important in the learning and the implementation of mathematics. To develop math skills, students are given the opportunity to use and practice the skills. Mathematical processes refer to skills in acquiring and applying mathematical knowledge, namely: a) reasoning; b) communication and connection; c) application and modeling; and d) thinking and heuristic skills. Metacognition refers to a person's awareness and ability to control their thought processes. To develop metacognitive awareness and strategies, and knowing when and how to use strategies, students are given the opportunity to solve non-routine and open-ended problems, discuss solutions, think hard and reflect on what is already being done, and monitor progress and make changes when it is necessary. While attitude refers to affective aspects in learning mathematics such as: beliefs about mathematics and its usefulness, interest and pleasure in learning mathematics, appreciation for the beauty and power of mathematics, confidence in using mathematics, and perseverance in solving problems.

Strands on the Mathematics Syllabus

The concepts and skills covered in the syllabus are organized along 3 strands. Developmental processes, metacognition and attitudes are embedded in the learning experiences associated with these strands. Table 1 shows the strands and substrands in the elementary school syllabus (1 s.d. 6).

Table 1. Strands on the elementary school syllabus (1 s.d. 6) [9]

Syllabus	Strands	Substrands	Descriptions
The First Elementary	Numbers and algebra	Integer	• Numbers up to 100 • Addition and subtraction • Multiplication and division
		Money	Money

Syllabus	Strands	Substrands	Descriptions
	Measurement and geometry	Measurement	• Lenght • Time
		Geometry	• 2D Figure
	Statistics	Data representation and interpretation	Graphic images
The Second Elementary	Numbers and algebra	Integer	• Numbers up to 100 • Addition and subtraction • Multiplication and division
		Fractions	• Integer fractions • Addition and subtraction
		Money	Money
	Measurement and geometry	Measurement	• Length, mass, and volume • Time
		Geometry	• 2D Figure • 3D Figure
	Statistics	Data representation and interpretation	Graphic image with scale
The Fourt Elementary	Numbers and algebra	Integer	• Numbers up to 100 • Addition and subtraction • Multiplication and division
		Fractions	• Integer fractions • Addition and subtraction
		Money	Money
	Measurement and geometry	Measurement	• Length, mass, and volume • Time
		Area and volume	Area dan perimeter
		Geometry	• Angle • Perpendicular and parallel lines
	Statistics	Data representation and interpretation	Bar Graph
The fourth Elementary	Numbers and algebra	Integer	• Numbers up to 100,000 • Factors and multiplications • Four operations
		Fractions	• Mixed numbers and irregular fractions • Fractions and sets of an object • Addition and subtraction
		Decimal	• Decimal to 3 digits behind the comma • Addition and subtraction • Multiplication and division
		Measurement	Time

Syllabus	Strands	Substrands	Descriptions
	Measurement and geometry	Area and volume	Area and perimeter
		Geometry	• Angle • Rectangle and square • axis of symmetry
	Statistics	Data representation and interpretation	Tables and line graphs
The fifth Elementary	Numbers and algebra	Integer	• Numbers up to 10.000.000 • Four operations
		Fractions	• Fractions and division • Four operations
		Decimal	Four operations
		Percentage	Percentage
		Ratio	Ratio
		Rate and speed	Rate
	Measurement and geometry	Area and volume	• Area of Triangle • Volume of cubes and blocks
		Geometry	• Angle • Triangle • Parallelogram, Rhombus, and trapezium
	Statistics	Data Analysis	• Average of the data
The sixth Elementary	Numbers and algebra	Fractions	Four operations
		Percentage	Percentage
		Ratio	Ratio
		Rate and speed	Distance, time and speed
		Algebra	Algebra
	Measurement and geometry	Area and volume	• Area and circumference of the circle • Volume of cubes and blocks
		Geometry	• Special quadrilateral • Nets
	Statistics	Data representation and interpretation	Circle diagram

Table 1 shows that at the elementary level the vastness and profunity of each substrands are given gradually from the first elementary to the sixth. In the first elementary syllabus integer substrands emphasize the operations of addition, subtraction, multiplication, and division up to the number 100. It comes up to In to the number 1000 at the second elementary, then up to the number 10,000 at the third elementary, up to 100,000 at the forth, and up to 100,000,000 at the fifth. At the second elementary, students start to learn fractions. The addition is done gradually on each syllabus. As well as the integer, the strands of measurement and geometry, at the first elementary it only focus on drawing graphs, then at the second one it is improved to drawing graphs with scales, at the third level it is deepened about bar graphs, then at the fourth level is deepened about tables and line graphs, next at the fifth level they learn the average of the data, and for the last level it is deepened about circle charts. Giving 3 strands gradually and tiered at the elementary level is aimed that students have a strong foundation, so that they can use the knowledge and experience they have when continuing in the next level, in which at the intermediate level.

There are 5 syllabuses in the intermediate mathematics curriculum, which fulfill the different needs, interests and abilities of students, namely: 1) Mathematics O; 2) Mathematics N(A); 3)

mathematics N(T); Additional Mathematics O; and 5) Additional Mathematics N(a). As in at the elementary level, in the intermediate level, the O, N (A), and N (T) Mathematics syllabus is focused on 3 strands, namely: numbers and algebra, geometry and measurement, and statistics and opportunity. While in additional mathematics O and N (A) are focused on algebra, geometry and trigonometry, as well as calculus. An overview of strands in the O and N (A) Mathematics syllabus can be seen in Table 2.

Tabel 2. Strands of mathematics on syllabus O and N (A) [8]

Strands	**Substrands code**	**Substrands**	**Description**
Numbers and algebra	N1	Number and its operation	Intermediate 1, 3/ 4
	N2	Ratio and proportion	Intermediate 1st , 2nd
	N3	Percentage	Intermediate 1st
	N4	Rate and speed	Intermediate 1st
	N5	Algebraic expressions and formulas	Intermediate 1st , 2nd
	N6	Functions and graphics	Intermediate 1st , 2nd , 3rd / 4th
	N7	Equations and inequalities	Intermediate 1st , 2nd , 3rd / 4th
	N8	Assembly language and notation	Intermediate 3rd / 4th
	N9	Matrix	Intermediate 3rd / 4th
Geometry and Measurement	G1	Angles, triangles, and triangles	Intermediate 1st
	G2	Congruence and congruence	Intermediate 2nd , 3rd / 4th
	G3	Spiral elements	Intermediate 3rd / 4th
	G4	Pythagorean theorem and trigonometry	Intermediate 2nd , 3rd 4th
	G5	Measurement	Intermediate 1st , 2nd , 3rd / 4th
	G6	Coordinate geometry	Intermediate 3rd / 4th
	G7	Two-dimensional vector	Intermediate 3rd / 4th
Statistics and opportunities	S1	Data handling and data analysis	Intermediate 1st , 2nd , 3rd / 4th
	S2	Opportunities	Intermediate 2nd , 3rd 4th

Table 2 shows that at the first Intermediate on strands geometry and measurement, students are provided with angles, triangles, and polygon, as well as measurements. At the second Intermediate, it is equipped with: concrete and congruence, Pythagorean theorem and trigonometry, and measurement. Meanwhile, at the third Intermediate, congruent and similarity, Pythagoras theorem, and measurements are re-studied, and it is supplemented with Circle elements, coordinate geometry, and two-dimensional vectors. In the strands of numbers and algebra, sets and notation, as well as new matrices are given at the intermediate level 3rd /4th. For an overview of the sub strands in the mathematics syllabus N (T) can be seen in Table 3.

Table 3. Distribution of substrands on N(T) Mathematics syllabus [7]

Strands	**Substrands code**	**Substrands**	**Description**
Numbers and algebra	N1	Number and its operation	Intermediate 1st, 3rd / 4th
	N2	Ratio and proportion	Intermediate 1st, 2nd, 3rd/ 4th
	N3	Percentage	Intermediate 1st
	N4	Rate and speed	Intermediate 2nd

	N5	Algebraic expressions and formulas	Intermediate 1st, 2nd, 3rd/ 4th
	N6	Functions and graphics	Intermediate 2nd, 3rd/ 4th
	N7	Equation	Intermediate 2nd, 3rd/ 4th
Geometry and measurement	G1	Angles, triangles and quadrilaterals	Intermediate 1st, 2nd
	G2	Symmetry	Intermediate 1st
	G3	Pythagorean Theorem	Intermediate 2nd
	G3	Trigonometry	Intermediate 3rd/ 4th
	G4	Measurement	Intermediate 1st, 2nd, 3rd/ 4th
Statistics and opportunities	S1	Data handling and data analysis	Intermediate 1st, 2nd, 3rd/ 4th

While the strands which ic for additional mathematics O and N (A) as well as pre-university level can be seen in Table 4.

Table 4. Strands of mathematics on additional syllabus O and N (A), as well as pre university level [2]–[6]

Level	**Silabus**	**Content**
Intermediate	Additional O and N (A) (Intermediate 3rd s.d. 4th)	• Algebra • Geometry and Trigonometry • Calculus
Pra University	H1	• Functions and graphics • Calculus • Opportunities and Statistics
	H2	• Functions and graphics • Lines and patterns • Vector • Introduction to complex numbers • Calculus • Opportunities and Statistics
	H2 addittional	• Algebra and calculus • Discrete Mathematics • Statistics and opportunities
	H3	• Function • Lines and patterns • Inequality • Number • Calculation

Approaches in Building Strands

Most of the Singapore's mathematics curriculum at the elementary, intermediate, and pre-university levels are hierarchical from easy to difficult, and simple to complex. The higher concepts and skills come later after the lower concepts and skills are given to the students and they has to be studied sequentially [2]–[8]. In addition, the syllabus design uses a spiral approach in building strands at all levels. Spiral approach is used in the design of syllabus from elementary school to pre-university [14]. At each level, the syllabus consists of several contents that facilitate the connection among the contents and they will be reviewed again at the next level with increasing profundity at each level. The use of a spiral approach allows students to reinforce mathematical concepts and skills that have been learned at previous levels [15]. The spriral approach allows fundamental concepts that have been learned in previous years to be relearned, expanded, deepened and integrated to a higher level [16].

Implementation

The success of the syllabus objectives at each level is inseparable from the implementation of the curriculum in the learning process. In the mathematics curriculum document issued by [9] the

selection of learning activities done uses the following 3 Principles, they are a) teaching to learn; b) teaching based on student interests; and c) learning which connects with everyday life, uses ICT, and emphasizes the achievement of 21st century competencies. Teaching is not only the transfer of knowledge, but teaching is the process of learning to understand, reason, apply, and problem solve. Moreover, it uses the experiences of the students to teach a material content. Then it uses a variety of media and give students the opportunity to be able to think, reason, and solve problems as well.

There are three learning phases used to implement the syllabus for each level, namely: readiness, engagement, and mastery phases [2]–[9]. The readiness phase is an important phase to understand the characteristics of students, both initial abilities, prerequisite knowledge that they already have, and the student's learning environment. The experience and learning environment are used to create a motivational context, so that understanding of the material studied will be easier. The next phase is involvement, in which the involvement is the involvement of students in the learning process. There are 3 recommended learning, namely: activity–based learning, discovery-based learning, and hands-on learning [2]–[9]. While the third phase is related to the mastery of students which includes: motivating practice activities, the use of reflective review, and extended learning [2]–[9].

In order to know the success level of students in the learning process, it can be measured by doing the assessment. According to [2]–[9], the assessments are in the form of formative and summative. The formative helps students to know and do the self reflection on the abilities that have been mastered or not, so that they can make improvement in mastering the materials before they take the summative. The formative is used to measure the achievement of the strands studied in the syllabus. The assessment focuses on: a) understanding of mathematical concepts; b) the ability in reasoning, communicating and connecting; c) the ability to formulate, represent, and solve problems; and d) the ability to develop strategies for solving non–routine problems [2]–[9]. In Beside formative and summative assessments, national examination assessments are also given in the last year to determine the mastery of competencies from each syllabus that have been studied. After spending six years to study at the elementary school, the students conduct a national examination, where the results of the exam help teachers and parents guide students in taking the appropriate program at the intermediate level [17].

Conclusion

The mathematics curriculum in Singapore focuses on problem-solving skills. Problem-solving abilities are developed by interrelated concepts, skills, thought processes, metacognition, and attitudes. Syllabus organizing contains 3 math concepts and skills that with learning experience (process, metacognition, and attitude). Most curriculum are hierarchies from easy to difficult, and simple to complex. In addition, the curriculum uses a spiral approach in building strands throughout the level.

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