THINKING PROCESS AND STRATEGY IN MATHEMATICAL PROBLEM SOLVING

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Abstract: The process of manipulating knowledge in a cognitive system that combines the knowledge already has in the long-term memory, or short-term memory with information obtained from experienced situations is called the thinking process. This process involves perception, abstraction, imagination, reasoning, and problem-solving. Facing problems and solving them is a life challenge that cannot be avoided. This article discusses the thought process in problem-solving as a conceptual study. The thought process includes receiving, processing, storing, and recalling information involving sensory registers, short-term memory (SHT) or working memory, and long-term memory (LTM). The thinking process in solving mathematical problems, namely (1) the stage of understanding and representing the problem which involves the process of receiving information, storing information and retrieving information; (2) the solution development stage, involving the process of obtaining information, processing information and storing information; (3) the stage of evaluating the answer, when the answer is considered, a thought process occurs, namely recalling information, processing information and storing information.

Keywords:Mathematical problems solving, Memory, Thinking processes

1. Introduction

Thinking is a transforming information process through the interaction of complex mental attributes to a new mental representations form. It is complicated because it consists of several processes: assessment, abstraction, reasoning, imagination, logic problem solving, concept formation, creativity, and intelligence (Solso et al., 2014). Cognitive processes occur when the thinking process occurs in a cognitive system that involves knowledge that has been stored in memory related to information from the situation being experienced (Utomo, 2015). It can be said that thinking is a mental activity that occurs when someone is encountering a problem. When dealing with a problem, there will be a complex interaction among several mental attributes such as perception, abstraction, imagination, reasoning, and problem-solving inside someone's mental.

Steps for problem-solving begin with identifying the problem, looking for ways to represent the problem, and then selecting actions that can be used to achieve goals. The goal is a solution to the problem at hand (Smith & Kosslyn, 2014). Solving math problems is a common task for all students at all levels of education. Problem-solving is the core goal of mathematics in school learning, where this skill is used in everyday human life as well as in the workplace (García, Boom, Kroesbergen, Núñez, & Rodríguez, 2019). Furthermore, problem-solving here is in situations that are not instant, in a standard or common way to reach a solution (Reiss & Törner, 2007).

Problem-solving skills are essential in mathematics in many countries around the world (Olivares, Lupiáñez, Segovia, & Lupiáñez, 2020). However, the 2018 PISA study results show that in Indonesia, only 1% of students achieve a grade at level 5 or higher in mathematics. This percentage value is still far from being compared to the OECD average of 11%. It shows that students still faced difficulties in modeling complex situations mathematically, selecting, comparing, and evaluating appropriate problem-solving strategies (OECD, 2018).

The discussion of the thought process in problem-solving is an interesting conceptual study to discuss. This study is expected to contribute to teachers, educators, and researchers to understand the thought processes that occur during mathematics problem-solving activities.

2. Discussion

2.1 Problem

A person often faces problems in his life. Facing problems and solving them is a life challenge that cannot be avoided. In the cognitive psychology context, a problem is when there is no instant, standard, or common way to achieve a goal. There are several types of problems, namely (1) problems whose initial state and purpose are clearly defined and there are binding rules called well-defined problems, (2) problems that are impossible to ascertain rules, initial states, operations, and even objectives are called ill define (Smith & Kosslyn, 2014). According to Matlin (2013), there are three components in every problem, namely the initial state, the goal state, and the obstacles.

The first component of the problem is the initial state. The initial state or the beginning condition is a state in which a person is currently facing a problem. The second component is the goal state, which is the desired state or the solution to the problem. Moreover, the last is the obstacle, which is a limitation that makes it difficult to proceed from the initial state to the goal state (Matlin, 2013). However, this limitation can be in the form of a series of workable operations that are actions or mental ones that a person can perform to get from the initial state towards the goal.

2.2 Problems in Mathematics

A problem is when a person is fully aware, and the problem is a challenge that cannot be solved immediately through existing routine procedures. For someone, the situation can be a problem, but for others, it is not necessarily a problem for him (Wahyudi & Anugraheni, 2017). There are various forms of problem presentation in mathematics in the form of irregular math problems, such as math story problems, describing mathematical numbers or events, illustrating pictures, or math puzzles. Furthermore, because these problems contain concepts in mathematics, they are called mathematical problems. Some of the criteria for problems in the form of word problems that are good in mathematics, as Hiebert (1997) explained, are as follows:

1. Involving and attracting student interest. Problems that attract students' interest include problems that come from the real world, relate to student interests¸ are even and attractive to all students¸ and require active student involvement.
2. Contains essential mathematical concepts. Mathematical problems are well connected with other problems or other subjects and mathematical concepts. Furthermore, aligning with the current mathematics curriculum.
3. Problems are open-ended and non-routine. Good math problems allow for multiple approaches and solutions. It is also not easy to solve using previously taught algorithms.
4. Problems are challenging but accessible to students. It takes students' persistence when solving the problem, but students also have sufficient knowledge of mathematics to understand the problem.
5. Problems are well made. It means that the problem contains clear and unambiguous words. Problems also reflect expectations and obtain assessable answers.

Although not all story problems are mathematical problems, through these story problems, the use of mathematics in real life can be seen (Van Dooren, Lem, De Wortelaer, & Verschaffel, 2019). Besides, mathematics problems can be in the form of non-routine problems that cannot be solved by algorithms but using heuristics. Various forms of mathematical problems become relevant and acceptable to solve.

Someone is said to have problems (Moursound, 2005) if they have the following four things: (1) understand the initial situation clearly defined; (2) understand the goals clearly (the desired final situation); (3) understand a set of resources that can be applied in helping to move from the initial situation given to the desired goal situation includes several things, namely time, knowledge, and skills; (4) having several resources, such as knowledge, skills, time, and energy, to achieve the desired final goal. So that if someone has these four things, they can continue towards solving the problem until they reach the expected goal, namely the solution to the problem.

2.3 Thinking Process in Mathematical Problem Solving

Thinking is closely related to memory, as described in Figure 1. The form of stimulation will be registered in the memory buffer, then transmitted to working memory, and only some information can enter long-term memory. Information can enter into long-term memory after going through the process of repetition and coding. In the memory structure, working memory gets two sources of information before producing output in the form of thoughts about the problem at hand.

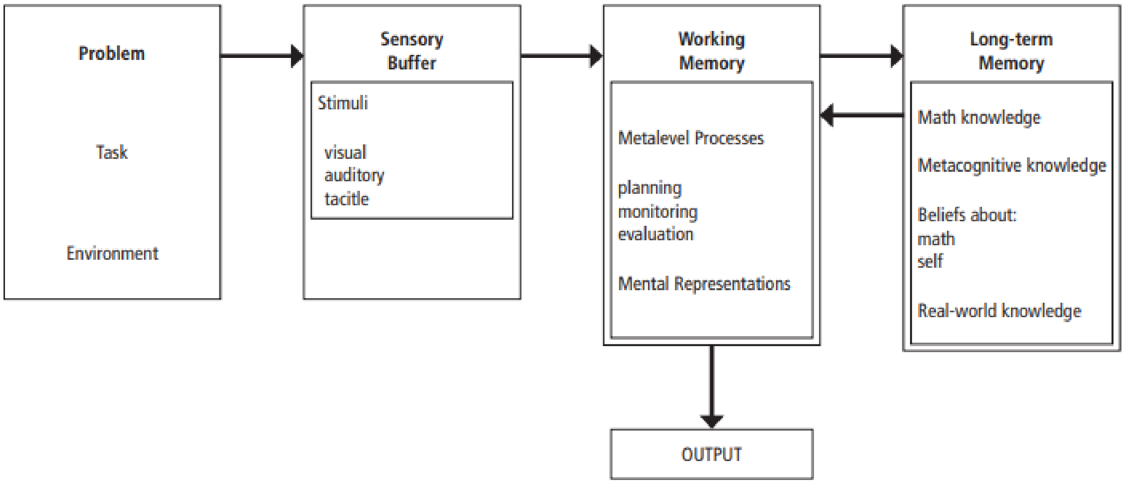

Figure 1. Memory Structure (Schoenfeld, 2016)

It begins with the idea of processing information, which is an action based on the coding of stimuli received through experience. A person's stimulation experiences can be visual, auditory, or tactile (concerning touch) listed in the sensory buffer section. If not ignored, then it is converted into working memory later in long-term memory. Sensory buffers, also known as iconic memories, can register much information, but only a few survive. Some information will be lost, and some will be transmitted to working memory. In this case, working memory receives information from two sources, namely sensory buffers and from long-term memory. "An extraordinary kind of intellectual skill, particularly in problem-solving, is called a cognitive strategy. In terms of modern learning theory, a cognitive strategy is a control process. An internal process by means of which thinking" (Gagne, 1970). Problem-solving is a special cognitive process involving internal processes in thinking. The process of manipulating knowledge in a cognitive system that combines the knowledge already held in long-term memory or short-term memory with information obtained from experienced situations is called the thinking process. Therefore, the thinking process is influenced by two important things, namely schemes and new knowledge. Schemes are initial knowledge in the form of mental structures that are owned by each individual. Usually, new knowledge can be obtained by a person when he interacts with the environment. The existing schema can be modified, added, or even replaced by the newly acquired knowledge. The process of assimilation and accommodation is included in the adaptation process. If the new experience obtained follows one's scheme, then the scheme is only modified or added through the assimilation process to create equilibrium. However, if the new experience gained is very different from the old scheme, the old scheme will be changed through the accommodation process.

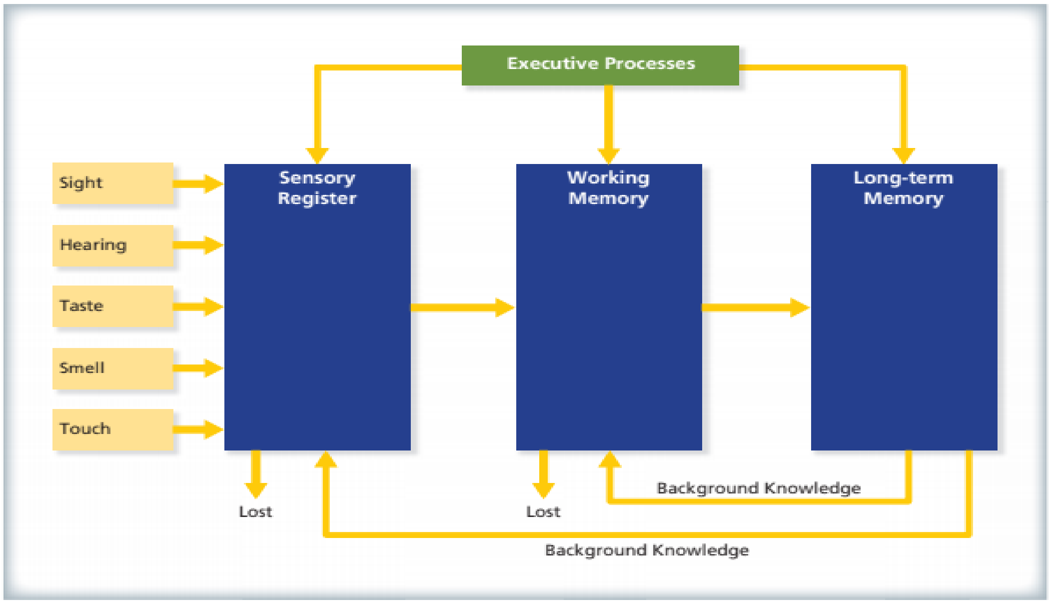

Figure 2. Information Processing Model (Slavin, 2018: 122)

The thinking process can be related to information processing theory, according to Gagne. In information processing theory, it is explained that mental processes are shown by someone to receive, process, store, and recall information for use. Like in Figure 2, Slavin (2018) explains that sensory registers receive information through external stimuli. There can be two possibilities from the amount of information received; namely, there is information that is forgotten, and there is also information that is passed on to short-term memory. The initial processing of information occurs when information is transferred from sensory recordings to short-term memory, also known as working memory. If the information already in the short-term memory undergoes repetition and encoding, the information will be transferred to the long-term memory for storage. However, the opposite can also occur in calling information that is already stored in long-term memory to be reconstructed and then concluded. By using information processing theory, the mental activities that occur can be detailed as follows:

1. Information reception, namely the entry of information in the form of external stimuli received by the senses, is included in the sensory register. Information that comes first is received through the sensory register. A large amount of information originating from the senses of sight, hearing, smell, taste, and touch) is held for a few seconds. If there is no further effort, this information will be lost. However, if certain information is given attention and given time, the information can be brought into working memory awareness (Slavin, 2018). The more senses involved in receiving information and the more meaningful the information obtained, the greater the tendency for the information to be received.
2. Information processing. An attempt is made so that information lasts and can be stored in memory is called information processing. The process occurs from the sensory register; then, it will undergo initial processing by involving the perception of information. Perception is defined as the interpretation of the stimuli that a person receives (Slavin, 2018). State of mind, past experiences, and motivation influence the initial processing of information. After initial processing, the information will be entered into short term memory or Short-Term Memory (STM). Information in the sensory register will be discarded if it is not pre-processed.
3. Information storage. If the information gets attention and time, the information will be able to enter the memory. Solso et al. (2008) argue that information that is understood and given attention by a person will be transferred to memory system components, namely Short-Term Memory (STM) and Long Term Memory (LTM). At any one time, there is an enormous amount of information stored in an information storage system, i.e., short term memory or working memory. In the end, only certain information enters working memory from the sensory register to long-term memory (LTM) through the repetition of information. Vast amounts of information can be stored in long-term memory (LTM), which is part of the memory system for an indefinite period. The storing (temporary) information occurs in the memory system, namely short-term memory (STM). The information received will be maintained at STM if there are repeated thoughts or pronouncements. There is an excellent chance that information can enter long-term

memory (LTM) if the working memory (STM) information occurs repetition. Repetition activities are often carried out in learning so that information can enter long-term memory. The process of storing information commonly occurs in LTM. There are three parts of long-term memory, namely episodic memory, semantic memory, procedural memory. Episodic memory is a place to store one's memories of personal experiences. Semantic memory is a place that stores facts and general knowledge in schematic form. Meanwhile, procedural memory is a place to store knowledge about doing something (Slavin, 2018). The network of interrelated ideas and can guide a person's understanding, and actions are called schemata. Information that fits into a well-developed schema is easier to learn than information that cannot be accommodated.

4. Information recall. There is much information stored in LTM. However, sometimes there is information from LTM called back to STM, which is often known by remembering. This process is called recalling information.

2.4 Strategies in Mathematical Problem Solving

The surest way to solve routine problems is with algorithms, where a series of procedures to solve an existing problem will sooner or later always yield the right answer. However, algorithms often take time and require large amounts of working memory and long-term memory. Meanwhile, the heuristic is a way to solve non-routine problems. It is used to solve problems, where some alternatives will be ignored and only explore alternatives that seem very likely to produce a solution.

Guidelines that are general steps in the problem-solving process in order to achieve the goal are called heuristics. Heuristics do not guarantee the correct solution, but only a guide to finding a solution. Meanwhile, the algorithm is a solution often used or commonly used and will produce the correct solution. Besides, the algorithm steps must be performed sequentially.

The types of problems range from mechanistic problems (routine problems) to logic problems (non-routine). However, it seems that, in general, the problem-solving process is neatly structured. Starting from what is expected, then the hypothesis of a possible solution is tested and confirmed; if not confirmed, a new hypothesis appears (Figure 3). This step is often known as trial and error.

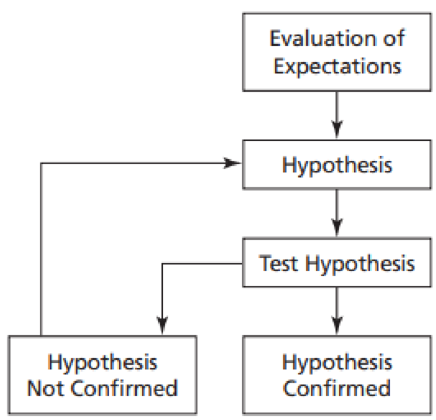

Figure 3. Problem-solving flow (Solso, 2014)

Meanwhile, the problem-solving process is studied from various sources, as shown in table 1. Some problem-solving steps, at the beginning it seems to start with understanding the problem, while Bransford & Stein (1993) start with problem identification and representation of the real problem as a series of activities in order to understand problems (Hayes, 1989). The next step is to plan a solution by choosing a strategy or method, then solving the problem as planned. The final part is to evaluate by looking back at the process passed to make sure that the solution obtained is the right solution for the purpose.

A person will solve a problem if he wants to achieve a particular goal, but the solution is not immediately found because he loses important information or is not clear how to achieve the goal (Matlin, 2013). However, in general, the troubleshooting steps can be described in several stages, such as the troubleshooting steps in table 1.

Table 1. Some problem-solving models

Polya (1973): Linear Model	Bransford & Stein (1993): IDEAL Model	Verschaffel et all (1999): Recursive Model
Understanding the problem	Identification of the problem	Build a mental representation of the problem. Heuristics: • Draw a picture. • Make a list, a scheme or a table • Distinguish relevant from irrelevant data • Use your real-world knowledge
Devising a plan	Definition of the problem	Decide how to solve the problem Heuristics: • Make a flowchart • Guess and check • Look for a pattern • Simplify the numbers
	Exploration of possible solutions	
Carrying out the plan	Acting according to the solution plan	Execute the necessary calculations
		Interpret the outcome and formulate an answer
Looking back	Look at effects of solutions	Evaluate the solution

Based on several existing problem-solving models, it can be summarized in three main phases. The first step is understanding the problem and representation of the problem at hand. The second is the development of solutions or solutions to problems (García et al., 2019). Next, the third is evaluating the answers. The thinking process in the three main stages of problem-solving is related to the thought process and indicators that can be developed, as described in table 2.

Table 2. Thinking process in solving mathematical problems

Mathematical Problem Solving	Thinking Process	Indicator (Utomo, 2015)
Understanding and representing the problem	Receiving information	Identify information from reading questions, pictures, tables, diagrams, or graphs on questions. Determine the core of the problem by stating and explaining the purpose of the problem.
	Storing information	Find information related to problems that do not exist in the questions. Make logical connections between the statements in the problem and pictures, tables, diagrams, or graphs in the form of arguments.
	Recalling information	Describe the questions using their sentences.
Developing Solution	Receiving information	Determine the core problem of the questions given.
	Processing information	Linking the knowledge they have with the information in the questions. Use thinking skills to come up with solutions.
	Storing information	Mention things related to the problem. Make logical arguments when finding a concept.
Evaluating Answer	Recalling information	Knowing the core of the problem from the questions given.
	Processing information	Use thinking skills to prove answers in different ways.
	Storing information	Generates logical arguments when checking the answer results.

When solving math problems, the reality in the field is the third stage, namely evaluating answers, is rarely done. Although an important role at this stage is to ensure that all process development solutions are correct and that all possible answers to the problem at hand have emerged.

3. Conclusion

The thought process includes receiving, processing, storing, and recalling information involving sensory registers, short-term memory (SHT) or working memory, and long-term memory (LTM). If it is related to solving mathematical problems, then this thinking process can be a sequential stage. Starting from the stage of understanding and representing the problem involves receiving information, storing information, and recalling information. It is followed by the second stage, namely, developing a solution. This second stage involves the process of receiving information, processing information, and storing information. The final stage is to evaluate the answers. When the answers are evaluated, a thought process occurs, namely recalling information, processing information, and storing information.

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